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Received April 2001
Accepted December 2001

Natural convection flow along a vertical circular cone with uniform surface temperature and surface heat flux in a thermally stratified medium

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Keywords *Natural convection, Laminar flow, Finite differences*

Abstract *The problem of laminar natural convection from a vertical circular cone maintained at either a uniform surface temperature or a uniform surface heat flux, and placed in a thermally stratified medium is considered. The governing non-similarity boundary layer equation for uniform surface temperature are analyzed by using two distinct solution methodologies; namely, (i) a finite difference method and (ii) a local non-similarity method. For uniform surface heat flux case, the solutions of the governing non-similarity boundary layer equations are obtained by using three distinct solution methodologies, namely, (i) a finite difference method, (ii) a series solution method and (iii) an asymptotic solution method. The solutions are presented in terms of local skin-friction and local Nusselt number for different values of Prandtl number and are displayed graphically. Effects of variations in the Prandtl number and stratification parameter on the velocity and temperature profiles are also shown graphically. Solutions obtained by finite difference method are compared with the other methods and found to be in excellent agreement.*

Nomenclature

B = Stratification rate
C_f = Skin friction
F = Dimensionless stream function
g = Acceleration due to gravity
Gr_x = Local Grashof number
Nu_x = Nusselt number
Pr = Prandtl number
r = Radius of the cone
T = Temperature of the fluid
T_w = Surface temperature
T_∞ = Temperature of the ambient fluid
$T_{\infty,0}$ = Ambient temperature at $x = 0$
u = Velocity component in the x-direction
v = Velocity component in the y-direction
x = Coordinate along a cone ray
y = Coordinate normal to cone surface

Greek symbols

α = Thermal diffusivity of fluid
β = Coefficient of thermal expansion
γ = Cone apex half-angle
η = Similarity variable
Φ = Dimensionless temperature function
κ = Thermal conductivity
ν = Kinematic viscosity

The authors are indebt to the reviewers for their valuable comments and constructive suggestion for present pice of work. They also expressed their heartiest indebtness to Dr. D. A. S. Rees (Unicversity of Bath, U. K) for helping in inproving the manuscript up to the present standard.

International Journal of Numerical Methods for Heat & Fluid Flow, Vol. 12 No. 3, 2002, pp. 290-305.

DOI 10.1108/09615530210422965

1. Introduction

Many free convection processes occur in environments with temperature stratification. Good examples are closed containers and environmental chambers with heated walls. Also of interest is free convection associated with heat rejection systems for long-duration deep ocean power modules where the ocean environment is stratified. Stratification of the fluid arises due to a temperature variation, concentration differences or the presence of different fluids.

Cheesewright (1967) presented a theoretical investigation of laminar free convection from a vertical plane in non-isothermal surroundings. He obtained similarity solutions of the governing equations dealing with various types of non-uniform ambient temperature distributions. Eichhorn (1969) studied the effect of linear thermal stratification on the heat transfer of a vertical plate and obtained solutions for three terms in the series expansions of the partial differential equations. Subsequently, Fujii *et al.* (1974) considered the effect of non-linear thermal stratification on the problem of (Eichhorn, 1969) and obtained solutions for four terms of the series expansion. Later, Chen and Eichhorn (1976) re-studied the problem of (Eichhorn, 1969) and obtained the solutions using the local non-similarity method developed by Sparrow *et al.* (1971); Minkowycz and Sparrow (1974). Yang *et al.* (1972) investigated the natural convection heat transfer from a non-isothermal vertical flat plate immersed in a thermal stratified medium. In their work, extensive numerical calculations based on similarity solutions had been carried out for a wide range of wall and ambient temperature distributions for Prandtl numbers between 0.1 and 20. Jaluria and Himasekhar (1983) studied the problem of natural convection flow in a plane thermal plume and flow over a heated vertical plate in an arbitrary, but stably stratified environment. In their paper, numerical solutions of the governing partial differential equations were obtained by finite difference methods for two values 6.7 and 0.7 of the Prandtl number, which correspond to water and air, respectively, at normal temperature. Kulkarni *et al.* (1986) investigated the problem of natural convection from an isothermal flat plate suspended in a linearly stratified fluid using the Von Karman–Pohlhausen integral solution method. Venkatachala and Nath (1981) studied the case of non-similar laminar natural convection in a thermally stratified fluid, taking into account the effect of mass transfer by using the implicit finite -difference scheme developed by Keller and Cebeci (1972). Later, Angirasa and Srinivasan (1989) presented a numerical study of the double-diffusive natural convection flow adjacent to a vertical surface in thermally stratified ambient. They considered situations where the two buoyant mechanisms aid as well as oppose each other. They studied the role of ambient thermal stratification by considering the simple case of linear temperature variation. Recently, the problem of Non-darcy free convection in a thermally stratified porous medium along a vertical plate with variable heat flux has been investigated by Hung and Chen (1997). They obtained the numerical solutions by implicit finite difference method.

Theoretical studies on laminar free convection flow on axisymmetric bodies have received wider attention, especially in case of non-uniform surface temperature and surface heat flux distributions. Mark and Prins (1953, 1954) developed the general relations for similar solutions on isothermal axisymmetric forms and showed that for the flow past a vertical cone has such a solution. Approximate boundary layer techniques utilized to obtain an expression for the dimensionless heat transfer. Braun *et al.* (1961) contributed two more isothermal axisymmetric bodies for which similar solutions exist, and used an integral method to provide heat transfer results for these and the cone over a wide range of Prandtl number. In the above investigation, the authors obtained the results by numerical integration of the differential equations for a fluid having Prandtl number 0.72. The similarity solutions for free convection from the vertical cone have been exhausted by Hering and Grosh (1962). They showed that the similarity solutions to the boundary layer equations for a cone exist when the wall temperature distribution is a power function of distance along a cone ray. In their paper they presented the results for isothermal surface as well as for the surface maintained at the temperature varying linearly with the distance measured from the apex of the cone for Prandtl number 0.7. Latter, Hering (1965) extended the analysis to investigate for low Prandtl number fluids. On the other hand, Roy (1974) has studied the same problem for high values of the Prandtl number. Na and Chiou (1979a) studied the effect of slenderness on the natural convection flow over a slender frustum of a cone. Later, Na and Chiou (1979b) studied the laminar natural convection flow over a frustum of a cone. In the above investigations the constant wall temperature as well as the constant wall heat flux were considered. On the other hand, Alamgir (1989) investigated the overall heat transfer in laminar natural convection flow from vertical cones by using the integral method. Recently, Hossain and Paul (2000a,b) have investigated the natural convection flow from a heated vertical permeable circular cone. The solutions were obtained against the local variable ξ that represents the streamwise distribution of the transpiration velocity.

Here, we investigate the effect of stratification on convection from a vertical circular cone with either a uniform surface temperature or a uniform surface heat flux, a topic which has not yet been discussed in the literature. The ambient temperature is taken as linear function of the distance measured from the apex of the cone. The governing non-similarity boundary layer equations for uniform surface temperature are analyzed by using two distinct solution methodologies; namely, (i) a finite difference method and (ii) a local non-similarity method. For uniform surface heat flux case, the solutions of the governing non-similarity boundary layer equations are obtained by using three distinct solution methodologies, namely, (i) a finite difference method, (ii) a series solution method and (iii) an asymptotic solution method. The solutions are presented in terms of local skin-friction and local Nusselt number for different values of Prandtl number and are displayed in graphically. Solutions obtained by finite difference method are compared with the other methods and found to be in excellent agreement. Effects of stratification parameter and Prandtl number on velocity and temperature profiles are also shown graphically, computed by only finite difference technique.

2. Mathematical formalism

Consider a steady two-dimensional laminar natural convection flow along a vertical cone with either a uniform surface temperature or a uniform surface heat flux, and which is immersed in a thermally stratified medium. The ambient temperature of the medium is $T(x)$ where x is the distance measured from the apex of the cone. The effect of viscous dissipation on thermal boundary layer is neglected. The physical coordinates (x, y) are chosen such that x is measured from the apex of the cone, O, in the stream wise direction and y is measured normal to the surface of the cone. The coordinate system and the flow configuration are shown in Figure 1.

Under the boundary layer approximations the flow is governed by the following boundary layer equations

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta\cos\gamma(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u, v are the fluid velocity components in the x- and y-directions, respectively, ν is the kinematic coefficient of viscosity, g is the acceleration due to gravity, β is the coefficient of volume expansion, α is the thermal diffusivity, γ is the cone apex half-angle and T is the temperature of the fluid.

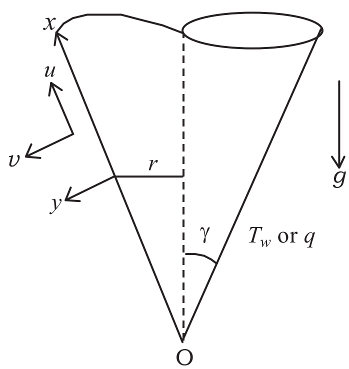

Figure 1. Physical model and co-ordinates system

As pointed out by Hering and Grosh (1962), several simplifications have incorporated into Equations (1)–(3). Under the assumption that the boundary layer is thin relative to the local cone radius, the local radius to a point in the layer has been replaced with the value at a cone surface, $r(x)$. Evidently, this condition is not satisfied in the neighborhood of the cone tip. Further, since the fluid-density difference, which is deriving force for natural convection has been replaced with the product $\beta(T - T_\infty)$, the equations are limited to small values of this term for liquids but arbitrary values of gases. Finally, because the pressure gradient across the boundary layer has been taken as negligible, the equations are strictly applicable to cones of small apex angles.

Complete definition of the problem requires specification of the boundary conditions, which are as follows

$$u = 0, \quad v = 0, \quad T = T_w \text{ and } q = -k\left(\frac{\partial T}{\partial y}\right) \text{ at } y = 0$$

$$u = 0, \quad T = T_\infty(x) = T_{\infty,0} + Bx \text{ as } y \to \infty \tag{4}$$

where T_w and q are the surface temperature and surface heat flux, respectively, $T_{\infty,0}$ is the ambient fluid temperature at the apex of the cone and $B(= \partial T_\infty/\partial x)$ is a constant which represents the stratification rate.

Case 1: Uniform surface temperature. We can introduce the following transformations

$$\psi = \nu r Gr_x^{1/4} f(\xi, \eta), \quad T - T_\infty(x) = (T_w - T_{\infty,0})\theta(\xi, \eta)$$

$$\eta = \frac{y}{x} Gr_x^{1/4}, \quad \xi = \frac{Bx}{T_w - T_{\infty,0}}, \quad Gr_x = \frac{g\beta \cos\gamma (T_w - T_{\infty,0}) x^3}{\nu^2}, \quad r = x \sin\gamma \tag{5}$$

where Gr_x is the local Grashof number, ξ is the dimensionless stratification parameter, η is the pseudo-similarity variable and ψ is the stream function defined by

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \text{ and } v = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{6}$$

Finally, the functions $f(\xi, \eta)$ and $\theta(\xi, \eta)$ are, respectively, the dimensionless stream function and the temperature function of the fluid in the boundary layer region.

Substituting the transformations given in (5) into (1)–(4), we obtained the following non-similarity system of equations

$$f''' + \frac{7}{4} ff'' - \frac{1}{2} f'2 + \theta = \xi\left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi}\right) \tag{7}$$

$$\frac{1}{\Pr}\theta'' + \frac{7}{4} f\theta' - \xi f' = \xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta\frac{\partial f}{\partial\xi}\right) \tag{8}$$

The corresponding boundary conditions to be satisfied are as given below:

$$\begin{aligned} &f = f' = 0, \quad \theta = 1 - \xi \ \text{at} \ \eta = 0 \\ &f' = 0, \quad \theta = 0 \ \text{as} \ \eta \to \infty \end{aligned} \tag{9}$$

where Pr $(= \nu/\alpha)$ is the Prandtl number.

For $\xi = 0$, the Equations (7) and (8) subjected to the boundary condition (9) have been solved by Hossain and Paul (2000a) for a non-isothermal surface.

For the present case, $\xi < 1$ is the practical limit for the upward boundary layer flow. When $\xi > 1$, buoyancy forces act downwards as the ambient fluid is hotter than the heated surface. Because of this characteristic we present results only for $\xi < 1$.

Solutions of the local non-similar partial differential Equations (7)–(8) subject to the boundary conditions (9) are obtained by using the implicit finite difference method known as the Keller-box scheme developed by Keller and Cececi (1972). This method has also been used recently by Hossain and Paul (2000a,b), Hossain *et al.* (2000c), Hossain and Takhar (1996). The equations are also solved by local non-similarity solution (LNS) procedure developed by Sparrow *et al.* (1971), Minkowycz and Sparrow (1974).

Once we know the values of the functions f and θ and their derivatives, it is important to calculate the values of the local skin-friction and local Nusselt number from the following relations:

$$\frac{1}{2} C_{fx} Gr_x^{1/4} = f''(\xi, 0) \tag{10}$$

$$\frac{Nu_x}{Gr_x^{1/4}} = -\theta'(\xi, 0) \tag{11}$$

Results obtained by the methods mentioned above are presented graphically in Figures 2(a) and (b) for different values of Prandtl number (i.e. Pr = 0.01, 0.1, 0.70, 7.0).

Case 2: Uniform surface heat flux. For natural convection flow along a vertical cone with uniform surface heat flux, the following transformations may be introduced

$$\psi = \nu r Gr_x^{1/5} F(\xi, \eta), \ T - T_\infty(x) = \frac{qx}{\kappa} Gr_x^{-1/5} \Phi(\xi, \eta)$$

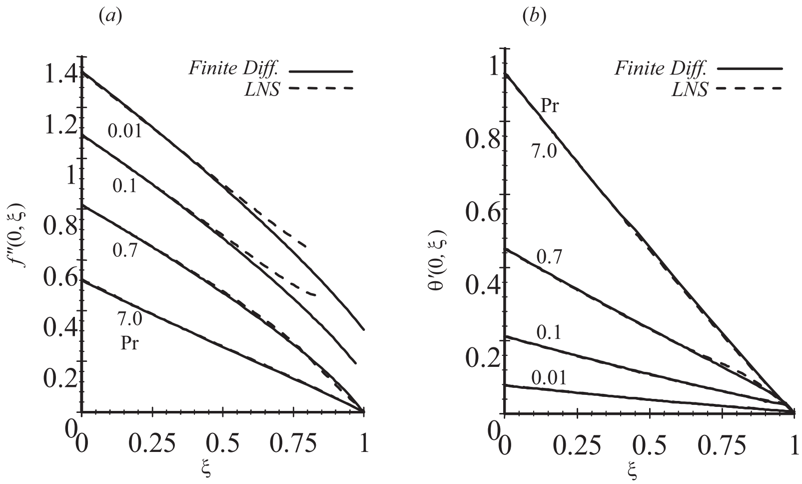

Figure 2.
(a) Local skin friction and (b) Local Nusselt number for different values of the Prandtl number against stratification parameter ξ for the uniform surface temperature case

$$\eta = \frac{y}{x} Gr_x^{1/5}, \quad \xi = \frac{B\kappa}{q} Gr_x^{1/5} = \frac{(T_\infty - T_{\infty,0})\kappa}{qx} Gr_x^{1/5}$$

$$Gr_x = \frac{g\beta \cos \gamma q x^4}{\kappa \nu^2}, \quad r = x \sin \gamma \tag{12}$$

where Gr_x is the local Grashof number, ξ is the dimensionless stratification parameter, η is the pseudo-similarity variable. $F(\xi,\eta)$ and $\Phi(\xi,\eta)$ are, respectively, the dimensionless streamfunction and temperature of the fluid in the boundary layer region. ψ is the streamfunction defined in Equation (6).

Substituting the transformation (18) into (1)–(4), yields the following non-similar system of equations

$$F''' + \frac{9}{5}FF'' - \frac{3}{5}F'2 + \Phi = \frac{4}{5}\xi\left(F'\frac{\partial F'}{\partial \xi} - F''\frac{\partial F}{\partial \xi}\right) \tag{13}$$

$$\frac{1}{\Pr}\Phi'' + \frac{9}{5}F\Phi' - \frac{1}{5}F'\Phi - \xi f' = \frac{4}{5}\xi\left(F'\frac{\partial \Phi}{\partial \xi} - \Phi'\frac{\partial F}{\partial \xi}\right) \tag{14}$$

and the corresponding boundary conditions transform to

$$F = F' = 0, \;\; \Phi' = -1 \;\text{ at }\; \eta = 0$$

$$F' = 0, \;\; \Phi = 0 \;\text{ as }\; \eta \to \infty \tag{15}$$

where Pr $(= \nu/\alpha)$ is the Prandtl number.

For $\xi = 0$, Equations (19) and (20) subject to the boundary condition (21) were solved by Hossain and Paul (2000b) for non-isothermal surfaces.

Solutions of the locally non-similar partial differential Equations (19) and (20) subject to the boundary conditions (21) are obtained using the Keller-box elimination technique. The values of the local skin-friction and local Nusselt number from the following relations:

$$\frac{1}{2}C_f Gr_x^{1/5} = F''(\xi, 0) \tag{16}$$

$$\frac{Nu_x}{Gr_x^{1/5}} = \frac{1}{\Phi(\xi, 0)} \tag{17}$$

Results obtained by this method are presented graphically in Figures 3(a) and (b) for different values of Prandtl number (i.e. $\Pr = 0.01,\ 0.1,\ 0.70,\ 7.0$). Below, we present the solutions valid for the small stratification parameter ξ as well as for the large stratification parameter ξ.

2.1 Solution for small values of ξ

Since near the apex of the cone, ξ is small for small x or small B or both, series solution of Equations (19) and (20) may be obtained by using perturbation method treating ξ as a perturbation parameter. Hence, we expand the functions $F(\xi,\eta)$ and $\Phi(\xi,\eta)$ in powers of ξ, that is, we take

$$F(\xi, \eta) = \sum_{i=0}^{\infty} \xi^i F_i(\eta) \text{ and } \Phi(\xi, \eta) = \sum_{i=0}^{\infty} \xi^i \Phi_i(\eta) \tag{18}$$

Substituting the above expansion into Equations (19) and (20) and equating the various powers of ξ up to $O(\xi)$, we obtain the following sets of equation

$$F_0''' + \frac{9}{5}F_0 F_0'' - \frac{3}{5}F_0'^2 + \Phi_0 = 0 \tag{19}$$

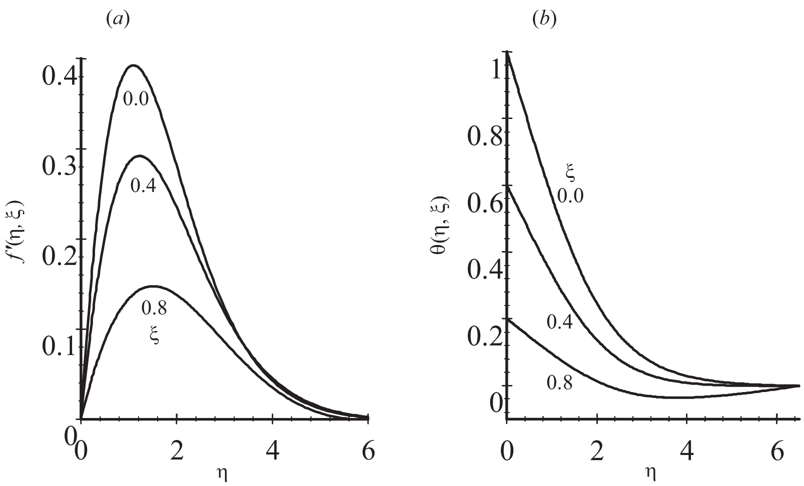

Figure 3.
(a) Velocity profile and (b) Temperature profile for different values of stratification parameter ξ while Prandtl number $\Pr = 0.7$ for the uniform surface temperature case. The curves for $\Pr = 0.7$ at $\xi = 0.0$ are due to Hossain and Paul (Hossain and Paul, 2000a)

$$\frac{1}{\Pr}\Phi_0'' + \frac{9}{5}F_0\Phi_0' - \frac{1}{5}F_0'\Phi_0 = 0 \tag{20}$$

$$F_0(0) = F_0'(0) = 0, \quad \Phi_0'(0) = -1$$

$$F_0'(\infty) = 0, \quad \Phi_0(\infty) = 0 \tag{21}$$

$$F_1''' + \frac{9}{5}F_0F_1'' + \frac{7}{5}F_0''F_1 - 2F_0'F_1' + \Phi_1 = 0 \tag{22}$$

$$\frac{1}{\Pr}\Phi_1'' + \frac{9}{7}F_0\Phi_1' + \frac{13}{5}\Phi_0'F_1 - F_0'\Phi_1 - \frac{1}{5}\Phi_0F_1' - f_0' = 0 \tag{23}$$

$$F_1(0) = F_1'(0) = 0, \quad \Phi_1'(0) = 0$$

$$F_1'(\infty) = 0, \quad \Phi_1(\infty) = 0 \tag{24}$$

The coupled Equations (25) and (26) are nonlinear, whereas (28) and (29) are linear. These equations are solved pair-wise one after another using the implicit Runge-Kutta-Butcher (Butcher (1974)) initial value solver together with the Nachtsheim-Swigert (Nachtsheim and Swigert (1965)) iteration scheme. Thus solutions are obtained for f_i and θ_i $(i = 0, \; 1)$ and their derivatives.

Knowing the value of F_i and Φ_i for $i = 0$, 1 and their derivatives, we can calculate the local skin-friction coefficient and the heat transfer from the following expressions

$$\frac{1}{2}C_f Gr_x^{1/5} = F''(\xi, 0) = F_0''(0) + \xi F_1''(0) \tag{25}$$

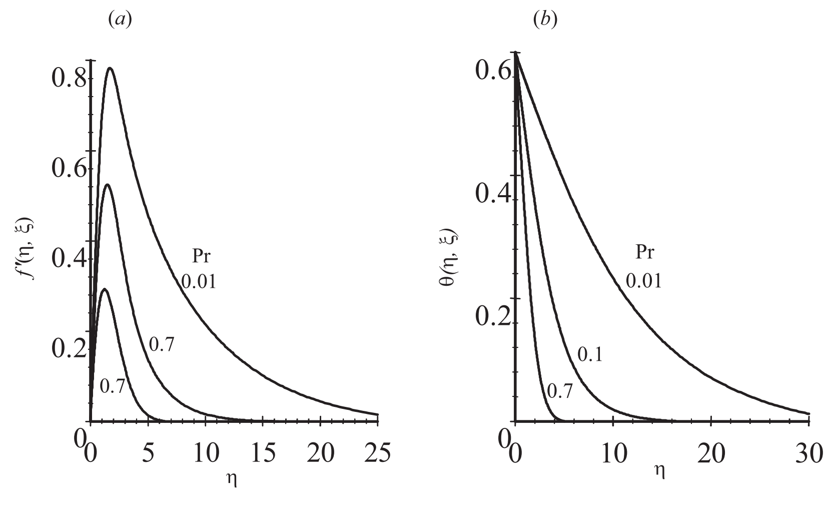

Figure 4.
(a) Velocity profile and (b) Temperature profile for different values of Prandtl number while stratification parameter $\xi = 0.4$ for the uniform surface temperature case

$$\frac{Nu_x}{Gr_x^{1/5}} = \frac{1}{\Phi(\xi, 0)} = 1/[\Phi_0(0) + \xi\Phi_1(0)] \tag{26}$$

The resulting values of local skin-friction and local Nusselt number for different values of the Prandtl number Pr (= 0.01, 0.1, 0.7, 7.0) are depicted in Figures 5(a) and (b). These results are compared with the corresponding values obtained from finite difference solution.

2.2 Solution for large values of ξ

In this section attention has been given to the behaviour of the solution to Equations (19) and (20) when ξ is large. An order of magnitude analysis of the various terms in these equations shows that the largest is $\xi F'$, in (20). This term has to be balanced and the only way to do this is to assume that, η is small and hence η-derivatives are large. Given that $\Phi = O(\xi^{-1})$ as $\xi \rightarrow \infty$, it is necessary to find the appropriate scaling F, Φ and η. On balancing the F''', Φ terms in (19) and Φ'', $\xi F'$ in (20), it is found that $\eta = O(\xi^{1/4})$, $F = O(\xi^{-1})$ and $\Phi = O(\xi^{-1/4})$ as $\xi \rightarrow \infty$. Therefore, the following substitutions are made

$$F = \xi^{-1}\tilde{F}(\xi, \tilde{\eta}), \quad \Phi = \xi^{-1/4}\tilde{\Phi}(\xi, \tilde{\eta}), \quad \tilde{\eta} = \xi^{1/4}\eta \tag{27}$$

Substituting this transformation into Equations (19) and (21), we get the following equations

$$\tilde{F}''' + \tilde{\Phi} + \frac{4}{5}\xi^{-5/4}\tilde{F}\tilde{F}'' = \frac{4}{5}\xi^{-1/4}\left(\tilde{F}'\frac{\partial \tilde{F}'}{\partial \xi} - \tilde{F}''\frac{\partial \tilde{F}}{\partial \xi}\right) \tag{28}$$

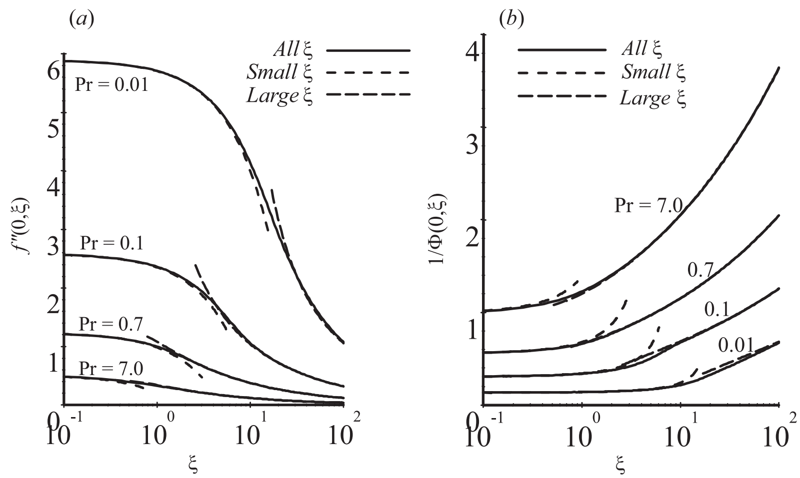

Figure 5.
(a) Local skin friction and (b) Local Nusselt number for different values of Prandtl number against stratification parameter ξ for the uniform surface heat flux case

$$\frac{1}{\Pr}\tilde{\Phi}'' - \tilde{f}' + \frac{4}{5}\xi^{-5/4}\tilde{F}\tilde{\Phi}' = \frac{4}{5}\xi^{-1/4}\left(\tilde{F}'\frac{\partial\tilde{\Phi}}{\partial\xi} - \tilde{\Phi}'\frac{\partial\tilde{F}}{\partial\xi}\right) \tag{29}$$

The corresponding boundary conditions are

$$\tilde{F}(\xi,0) = \tilde{F}'(\xi,0) = 0, \;\; \tilde{\Phi}'(\xi,0) = -1$$

$$\tilde{F}'(\xi,\infty) = 0, \;\; \tilde{\Phi}(\xi,\infty) = 0 \tag{30}$$

where primes denote differentiation with respect to $\tilde{\eta}$.

Since ξ is large, the functions $\tilde{F}(\xi,\tilde{\eta})$ and $\tilde{\Phi}(\xi,\tilde{\eta})$ are expanded in a power series in negative powers of ξ, that is, we take

$$\tilde{F}(\xi,\tilde{\eta}) = \sum_{i=0}^{\infty}\xi^{-5/4i}\tilde{F}_i(\tilde{\eta}) \text{ and } \tilde{\Phi}(\xi,\tilde{\eta}) = \sum_{i=0}^{\infty}\xi^{-5/4i}\tilde{\Phi}_i(\tilde{\eta}) \tag{31}$$

Now substitution of the above expansion into Equations (34) to (36) and equating of the coefficients of various powers of ξ up to $O(\xi^{-5/4})$ yields the following equations

$$\tilde{F}_0''' + \tilde{\Phi}_0 = 0 \tag{32}$$

$$\frac{1}{\Pr}\tilde{\Phi}_0'' - \tilde{f}_0' = 0 \tag{33}$$

$$\tilde{F}_0(0) = \tilde{F}_0'(0) = 0, \tilde{\Phi}_0'(0) = -1$$

$$\tilde{F}_0'(\infty) = 0, \tilde{\Phi}_0(\infty) = 0 \tag{34}$$

$$\tilde{F}_1''' + \frac{4}{5}\tilde{F}_0\tilde{F}_0'' + \tilde{\Phi}_1 = 0 \tag{35}$$

$$\frac{1}{\Pr}\tilde{\Phi}_1'' + \frac{4}{5}\tilde{F}_0\tilde{\Phi}_0' - \tilde{F}' = 0 \tag{36}$$

$$\tilde{F}_1(0) = \tilde{F}_1'(0) = 0, \;\; \tilde{\Phi}_1'(0) = 0$$

$$\tilde{F}_1'(\infty) = 0, \;\; \tilde{\Phi}_1(\infty) = 0 \tag{37}$$

The solutions of Equations (38) to (43) are

$$F_0''(\xi,0) = \frac{1}{\sqrt{\Pr}} \tag{38}$$

$$\Phi_0(\xi,0) = \frac{\sqrt{2}}{\Pr^{1/4}} \tag{39}$$

$$F''_1(\xi, 0) = \frac{6(1 - 11\mathrm{Pr})}{75\sqrt{2}}\mathrm{Pr}^{-7/4} + \frac{16}{25\sqrt{2}}\mathrm{Pr}^{-3/4} \tag{40}$$

$$\Phi_1(\xi, 0) = \frac{4(1 - 11\mathrm{Pr})}{75}\mathrm{Pr}^{-3/2} + \frac{12}{25}\mathrm{Pr}^{-1/2} \tag{41}$$

Thus the local skin-friction and local Nusselt number are as follows

$$\frac{1}{2}C_f Gr_x^{1/5} = F''(\xi, 0) = \xi^{-1/2}\left[\frac{1}{\sqrt{\mathrm{Pr}}} + \xi^{-5/4}\left\{\frac{6(1 - 11\mathrm{Pr})}{75\sqrt{2}}\mathrm{Pr}^{-7/4} + \frac{16}{25\sqrt{2}}\mathrm{Pr}^{-3/4}\right\}\right] \tag{42}$$

$$\frac{Nu_x}{Gr_x^{1/5}} = \frac{1}{\Phi(\xi, 0)} = \xi^{1/4}\Big/\left[\frac{\sqrt{2}}{\mathrm{Pr}^{1/4}} + \xi^{-5/4}\left\{\frac{4(1 - 11\mathrm{Pr})}{75}\mathrm{Pr}^{-3/2} + \frac{12}{25}\mathrm{Pr}^{-1/2}\right\}\right] \tag{43}$$

The asymptotic solutions obtained from the above expressions for different values of Prandtl number are compared with the solution of the finite difference method in Figure 5(a) and (b).

3. Results and discussion

In the present paper, we have investigated the problem of laminar natural convective flow and heat transfer from a vertical circular cone immersed in a thermally stratified medium with either a uniform surface temperature or a uniform surface heat flux. For the case of a uniform surface temperature, the solutions are obtained by means of a finite difference method and the local non-similarity method. The solutions, for uniform surface heat flux case, obtained by using: *first*, the finite difference method for all ξ, *second*, the perturbation method for small ξ and, *third*, the asymptotic method for large ξ, of the momentum and energy equations. The results are presented in terms of the local skin-friction, local Nusselt number, velocity profile and temperature profile. The comparison between the finite difference solutions to the solutions by other methods is found to be excellent.

3.1 Uniform surface temperature

The numerical values of local skin-friction, $C_{fx}Gr_x^{1/4}/2$ and local Nusselt number, $Nu_x/Gr_x^{1/4}$, against stratification parameter ξ for different values of Prandtl number $\mathrm{Pr}\ (= 0.01,\ 0.1,\ 0.7,\ 7.0)$ are displayed in Figures 2(a) and (b) respectively. These figures show that the results obtained by the finite difference method and the local non-similarity method are in excellent agreement. From Figures 2(a) and (b) we observe that both the local skin-friction and local

Nusselt number decrease with increases in ξ. Figure 2(a) shows that an increase in the value of Prandtl number, Pr, leads to a decrease in the value of local skin-friction. On the other hand, from Figure 2(b) it can be observed that the value of local Nusselt number increases with the increasing values of Prandtl number.

To observe the effect of stratification parameter on the dimensionless velocity profile, $f'(\xi, \eta)(= ux/\nu Gr_x^{1/2})$, and the dimensionless temperature profile, $\theta(\xi, \eta)\,[=(T - T_\infty(x))/(T_w - T_{\infty,0})]$, in the flow field, computed only by the finite difference method. The numerical values of the dimensionless velocity and temperature distributions are shown graphically in figure 3(a) and (b), respectively, against η at $\xi = 0.0, 0.4, 0.8$ while Pr $= 0.7$. From Figure 3(a) and (b), we observe that the fluid velocity and temperature profiles decrease with increasing values of the stratification parameter ξ. In figure 3(a), it can also be observed that at each value of ξ there exist local maxima in the velocity profile within the boundary layer region. These maximum values are 0.39268, 0.29193 and 0.14746 at $(\eta, \xi) = (1.09948,\ 0.0)$; (1.22203, 0.4) and (1.50946,0.8), respectively. In Figure 3(b), negative values of non-dimensional temperature appear in the 'wings' of the profile. This occurrence is often referred to as 'temperature defect'.

The effects of varying the Prandtl number, Pr $(= 0.01,\ 0.1,\ 0.7)$, on the dimensionless velocity, $f'(\xi, \eta)$ and the dimensionless temperature, $\theta(\xi, \eta)$, distributions against η at $\xi = 0.4$ are shown in Figures 4(a) and (b). Figures 4(a) and (b) show that when Prandtl number increases, then boththe velocity profile and temperature profile decreases. It can also be observed from the Figure 4(a) that for each value of Pr there exist local maxima in velocity profile within the boundary layer region. For Pr $= 0.01$, 0.1 and 0.7, the maximum values occur at $\eta = 1.67876, 1.47355$ and 1.22202, and they are 0.78296, 0.52460 and 0.29193, respectively. We further observe that, both the momentum and thermal boundary layer thicknesses decrease with the increasing values of Pr.

3.2 Uniform surface heat flux

The numerical values of local skin-friction, $C_f Gr_x^{1/5}/2$ and local Nusselt number, $Nu_x/Gr_x^{1/5}$, against stratification parameter ξ for different values of Pr $(= 0.01,\ 0.1,\ 0.7,\ 7.0)$ are depicted in Figures 5(a) and (b), respectively. From these figures it can be seen that the results for the series solution method as well as the asymptotic method are in excellent agreement with the finite difference solutions. Figure 5(a) shows that the values of local skin-friction, $C_f Gr_x^{1/5}/2$, decrease to the asymptotic value as ξ increases. With increasing values of the Prandtl number, Pr, it can be seen that the value of local skin friction decreases. From Figure 5(b) we observe that the value of local Nusselt number increases as ξ increases. This figure also shows that as Pr increases the value of local Nusselt number also increases.

Now attention is given to the effect of stratification parameter on the dimensionless velocity profile, $f'(\xi, \eta)\ (= ux/\nu Gr_x^{2/5})$ and the dimensionless temperature profile, $\theta(\xi,\ \eta)\ [= \kappa(T - T_\infty(x))\,Gr_x^{1/5}/qx]$, in the flow field, obtained by the finite difference method. The numerical values of dimensionless velocity and temperature distribution are depicted graphically in Figure 6(a) and (b), respectively, against η for values of $\xi = 0.0,\ 5.0,\ 10.0,\ 20.0$ while $\Pr = 0.7$. From Figure 6(a) it can be seen that the velocity profiles decrease with the increase in stratification parameter ξ. It can also be seen that at each value of ξ there exist local maximum values in velocity profile in the boundary layer region. These maximum values are 0.50547, 0.17134, 0.10423 and 0.06256 at $(\eta,\ \xi) = (0.94233,\ 0.0)$; (0.78384, 5.0); (0.67251, 10.0) and (0.56663, 20.0), respectively. Figure 6(b) shows that the temperature profiles decrease as the stratification parameter ξ increase. As in the case of isothermal surface, in the present case also negative values of non-dimensional temperature appear in the outer edge of the profile for higher values of ξ. The reason of this occurrence has already been explained in the earlier section. We further observe that the momentum and thermal boundary layer thickness decreases with the increasing values of ξ.

4. Conclusions

The present paper deals with the effect of stratification parameter, ξ, and Prandtl number, Pr, on laminar free convection boundary layer flow from a vertical circular cone with uniform surface temperature as well as with uniform surface heat flux. The governing non-similar boundary layer equations for the uniform surface temperature case are solved using a finite difference method and a local non-similarity method. For the uniform surface heat flux case, the solutions of the governing non-similar boundary layer equations are obtained by using three distinct solution methodologies, namely, (i) a finite difference method, (ii) a series solution method and (iii) an asymptotic solution method.

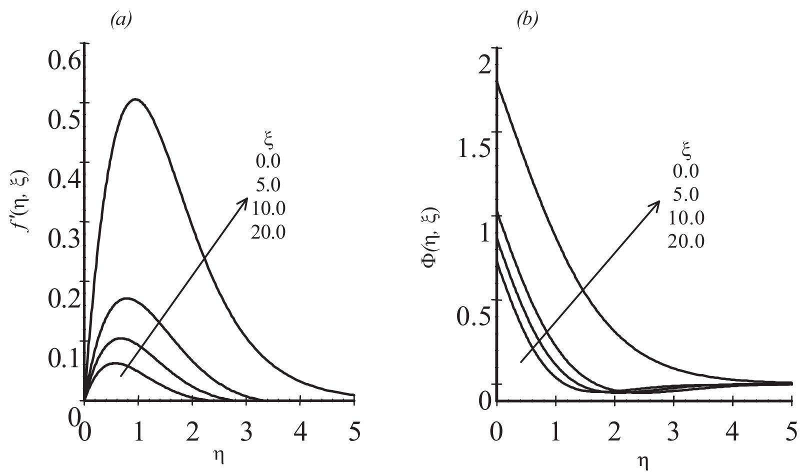

Figure 6. (a) Velocity profile and (b) Temperature profile for different values of stratification parameter ξ while Prandtl number $\Pr = 0.7$ for the uniform surface heat flux case. The curves for $\Pr = 0.7$ at $\xi = 0.0$ are due to Hossain and Paul (Hossain and Paul, 2000b)

We obtained the solutions by several methods to establish the accuracy of our solution. The simulated results are expressed in terms of local skin friction, local Nusselt number, velocity and temperature profile. From the present investigations, it may concluded that:

(1) For both the uniform surface temperature and uniform surface heat flux cones, the value of local skin-friction decreases with the increase in stratification parameter, ξ, on the other hand, the value of local Nusselt number decreases for the case of uniform surface temperature but for the uniform surface heat flux case its value increases.

(2) An increase in the value of Prandtl number, Pr, leads to decrease in the value of skin-friction coefficient but the value of local Nusselt number increases with the increasing values of Prandtl number, Pr, for both the uniform surface temperature and uniform surface heat flux cones.

(3) Both the velocity and the surface temperature of the fluid decreases due to the increase in the suction parameter, ξ, for both cases.

(4) With the increase in the value of Prandtl number, Pr, both the momentum and thermal boundary layer thickness decreases for uniform surface temperature cone.

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